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QCD JET BROADENING IN HADRON-HADRON COLLISIONS [1]

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Abstract

We present QCD predictions for the broadening of jets in hadron-hadron hard scattering with increasing transverse energy, E_T. The quantity studied is the total scalar transverse momentum perpendicular to the transverse thrust axis. This quantity is infrared-finite and asymptotically insensitive to soft hadronization effects. Our results imply substantially greater jet broadening than in e^+e^- annihilation. They also suggest, however, that higher-order QCD corrections to this quantity will be large.

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We present QCD predictions for the broadening of jets in hadron-hadron hard scattering with increasing transverse energy, E_T. The quantity studied is the total scalar transverse momentum perpendicular to the transverse thrust axis. This quantity is infrared-finite and asymptotically insensitive to soft hadronization effects. Our results imply substantially greater jet broadening than in e^+e^- annihilation. They also suggest, however, that higher-order QCD corrections to this quantity will be large.

Text

The properties of hadronic jets produced in hard collisions are of interest in their own right and also for the design of detectors at future accelerators such as the SSC. Although all these properties are predicted in principle by QCD, in practice it has been necessary to rely instead on Monte Carlo simulations1, which are based on leading-pole approximations to QCD combined with various models for the conversion of quarks and gluons into hadrons ("hadronization").

The leading pole approximation should be valid for parton distributions at small angles to the axis of a jet, and the Monte Carlo simulations do seem to be successful in this region. However, in many contexts secondary jets and tails of distributions associated with large-angle parton emission can be important. For such configurations the leading pole approximation is invalid and more exact perturbative calculations become necessary.

In this paper we present results of a full leading-order perturbative calculation of a quantity that probes the large-angle tails of jets, defined as follows. Consider a hard hadron-hadron collision which produces various outgoing hadrons k with transverse momenta $\vec{p}_t^{\,(k)}$. Neglecting hadronic masses, the total transverse energy is defined as,

$$E_T = \sum \left|\vec{p}_t^{\,(k)}\right| \tag{1}$$

The transverse thrust is,

$$T_t = Max \sum_k \left|\vec{p}_t^{\,(k)}.\vec{n}_t\right| / E_T \tag{2}$$

where the transverse thrust axis is the transverse unit vector $\vec{n}_t$ that achieves this maximum. The quantity we study is,

$$Q_t = \sum_k \left|\vec{p}_t^{\,(k)} \wedge \vec{n}_t\right| \tag{3}$$

which measures the total transverse momentum perpendicular to the transverse thrust axis. Our predictions are for $\langle Q_t \rangle$, the mean value of Q_t for all events with a given transverse energy E_T.

The quantity $\langle Q_t \rangle$ is of interest because:-

1. It probes an important aspect of jet structure and should be relatively easy to measure experimentally;
2. It is infrared-finite and, as discussed below, it should not be highly sensitive to soft hadronization effects. It should therefore be perturbatively calculable at sufficiently high E_T;
3. It is not calculable in the leading-pole approximation and therefore provides a test of the accuracy of leading-pole results outside their domain of strict validity. As mentioned above, this makes it especially useful as a test of Monte Carlo simulations.
4. The analogous quantity in e^+e^- annihilation is well described[2,3] by a perturbative calculation to lowest nontrivial order.

In perturbation theory, a non-zero value of Q_t first occurs in order g^6 from $(2 \to 3)$ parton subprocesses. For the process $1 + 2 \to a + b + c$, we define

$$z_i = \frac{2\left|\vec{p}_t^{\,(i)}\right|}{E_T}, \quad i = a, b, c \tag{4}$$

Then the transverse thrust axis is along the direction of the parton with the largest z_i,

$$T_t = Max\ z_i \tag{5}$$

and for the three parton final state Q_t is given by,

$$Q_t = \frac{2E_T}{T_t}\sqrt{(1-z_a)(1-z_b)(1-z_c)} \tag{6}$$

At the three parton level Q_t is proportional to the acoplanarity[5], defined as

$$A = Min \frac{\sum_k \left|\vec{p}^{(k)}.\vec{n}\right|}{\left|\vec{p}^{(k)}\right|} \tag{7}$$

However, for multiparton states the axis that minimizes A may differ from the transverse thrust axis.

To study the possible effects of parton fragmentation into hadrons, consider a simple model in which parton i fragments independently into a jet of hadrons $\{h\}$. Then we may write the transverse momentum of hadron h in the form

$$\vec{p}_t^{(h)} = x^{(h)}\vec{p}_t^{(i)} + \vec{q}_t^{(h)} \tag{8}$$

where

$$\vec{q}_t^{(h)}.\vec{p}_t^{(i)} = 0, \ \sum_h x^{(h)} = 1, \ \sum_h \vec{q}_t^{(h)} = 0 \tag{9}$$

For soft hadronization, $\vec{q}_t^{(h)}$ should be small. Expanding to first order in this quantity, we have

$$\sum_h \left|(\vec{p}_t^{(h)} \wedge \vec{n}_t\right| \approx \sum_h x^{(h)}\left|\vec{p}_t^{(i)} \wedge \vec{n}_t\right| + \sum_h \vec{q}_t^{(h)}.\vec{n}_t \frac{\vec{p}_t^{(i)}.\vec{n}_t}{\left|\vec{p}_t^{(i)} \wedge \vec{n}_t\right|} \tag{10}$$
$$= \left|\vec{p}_t^{(i)} \wedge \vec{n}_t\right|,$$

Thus to this order there is no hadronization correction to the value of Q_t. In practice, the energy dependence of the corresponding quantity for $e^+e^- \to$ hadrons does agree with the lowest-order perturbative prediction. However, the value obtained for Λ in lowest order depends somewhat on the hadronization model, corresponding to an uncertainty of about $\pm$ 12% in α_S at PETRA energies.[2,3]

If $f_A(x_1)$ and $f_B(x_2)$ represent the momentum fraction distributions of partons 1 and 2 in hadrons A and B colliding at c.m. energy $\sqrt{S}$, we find,

$$\frac{d\sigma}{dE_T} = \frac{E_T}{32\pi S^2}\int dy_a dy_b \frac{f_A(x_1)}{x_1}\frac{f_B(x_2)}{x_2}\overline{\sum}\left|M^{12\to ab}\right|^2, \tag{11}$$

$$\langle Q_t\rangle \frac{d\sigma}{dE_T} = \frac{E_T^4}{512\pi^4 S^2}\int dy_a dy_b dy_c dz_a dz_b \frac{f_A(x_1)}{x_1}\frac{f_B(x_2)}{x_2}\frac{z_a z_b z_c}{T_t}\overline{\sum}\left|M^{12\to abc}\right|^2 \tag{12}$$

where y_i is the rapidity of outgoing parton i.

In Fig. 1 we show the prediction for the differential cross-section $d\sigma/dE_T$ at c.m. energies of 0.63 and 1.8 TeV for $p\bar{p}$ collisions and 40 TeV for pp. The structure functions of Eichten et al.[6] (set 2, Λ=0.29 GeV) were used and the predictions are in good agreement with their work. Results obtained using the structure functions of Duke and Owens[7] (set 1, Λ = 0.2 GeV) are similar but slightly smaller owing in part to the smaller value of Λ. For all our predictions the scale of the running coupling constant is taken to be E_T.

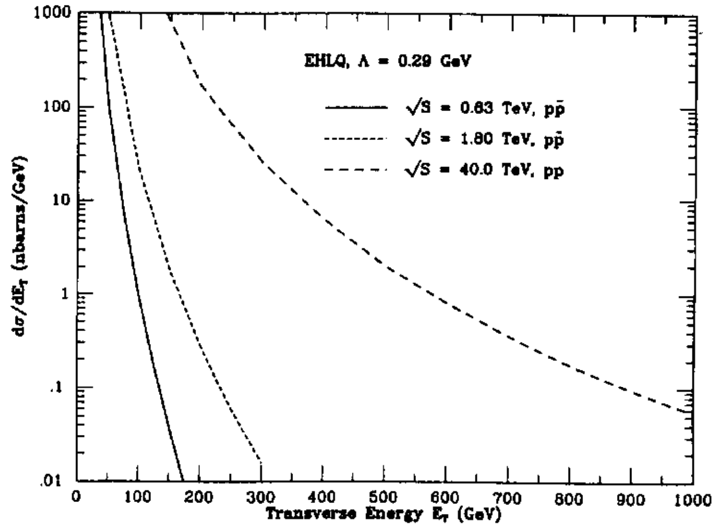

Fig. 1

Predicted differential cross section $d\sigma/dE_T$ vs. E_T.

Fig. 2 diplays the prediction for the mean value of Q_t as a function of E_T for $p\bar{p}$ collisions at $\sqrt{S}$ = 0.63 TeV, again using the structure functions of Eichten et al. The structure functions of Duke and Owens (set 1) lead to results which are about 10 % smaller. As shown by the dashed and dotted curves, gluon scattering is dominant at low values of E_T, becoming negligible only above 250 GeV. The value of $\langle Q_t\rangle$ is higher in gluon scattering than in quark scattering owing to the larger colour charge of the gluon. In the leading logarithm approximation the ratio would be C_A/C_F = 9/4 but in the full leading-order calculation it is not so large (about 1.3).

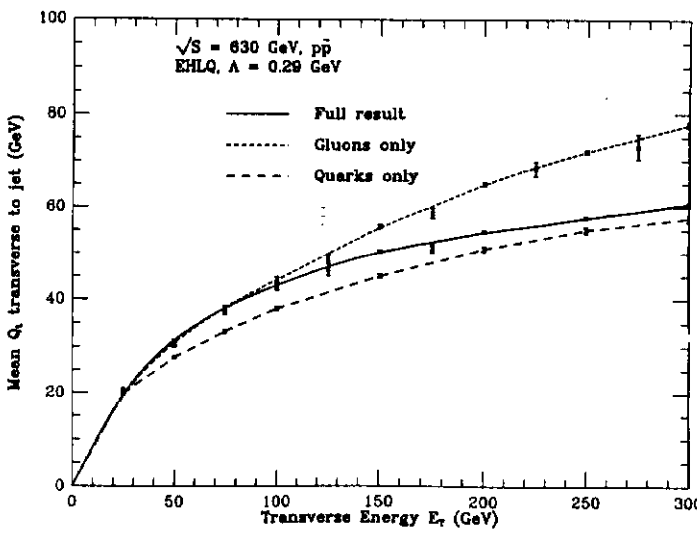

Fig. 2

Leading-order predictions for $\langle Q_t\rangle$ vs. E_T in $p\bar{p}$ collisions at c.m. energy $\sqrt{S}$ = 0.63 TeV. The "error bars" indicate the estimated uncertainties in the numerical integrations.

The corresponding quantity in e^+e^- annihilation[3] is the total scalar momentum transverse to the thrust axis, which at a c.m. energy of 30 GeV is about 8 GeV. We find larger values in hadron-hadron collisions, even when quark scattering is dominant, owing to the additional gluon radiation from initial state partons.

Figs. 3 and 4 show the full leading-order predictions for $\langle Q_t \rangle$ in $p\bar{p}$ collisions at $\sqrt{S} = 1.8$ TeV and pp collisions at 40 TeV. The values obtained at these enegies as well as at $\sqrt{S} = 0.63$ TeV are very large: so large, in fact as to be unphysical at low values of E_T. The kinematic upper limit on $\langle Q_t \rangle / E_T$ is 0.71 for an arbitrary final state. The kinematic limit for a three parton final state can be derived from Eq.(6) and is 0.58. As we show in Fig. 5, these bounds are exceeded in a range of relative transverse energies x_T that shrinks with increasing S.

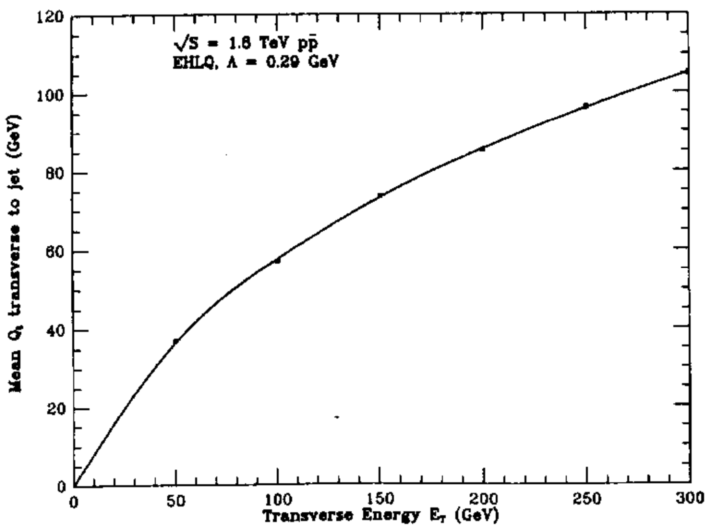

Fig. 3

As in Fig. 2, but for $\sqrt{S} = 1.8$ TeV.

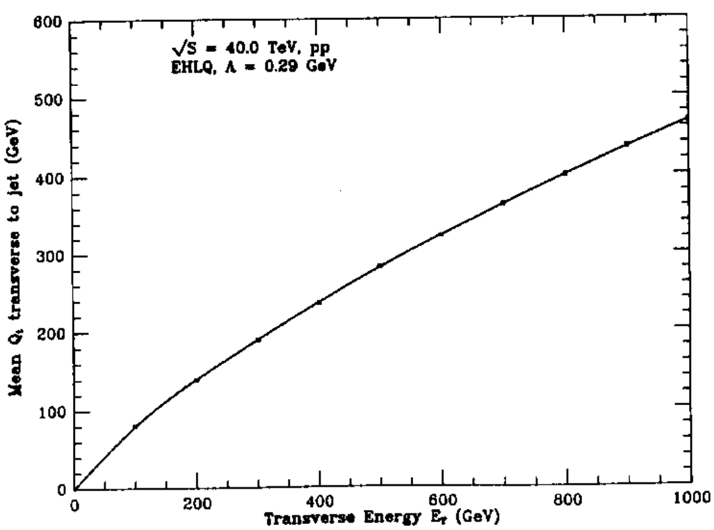

Fig. 4

As in Fig. 2, but for pp collisions at $\sqrt{S} = 40$ TeV.

The leading-order perturbative prediction of $\langle Q_t \rangle$ is not bound to lie within kinematic limits, because it is obtained by dividing a three-parton observable [Eq. (12)] by the two-parton cross section [Eq. (11)]. In general, the prediction of $\langle Q_t \rangle$ to order α_S^n involves states of up to $(n+2)$ partons in the numerator but only $(n+1)$ in the denominator, so the result may be unphysical to any fixed order. However, the full sum to all orders would involve the same states in numerator and denominator and would therefore necessarily respect kinematical bounds, even if QCD were not the correct theory.

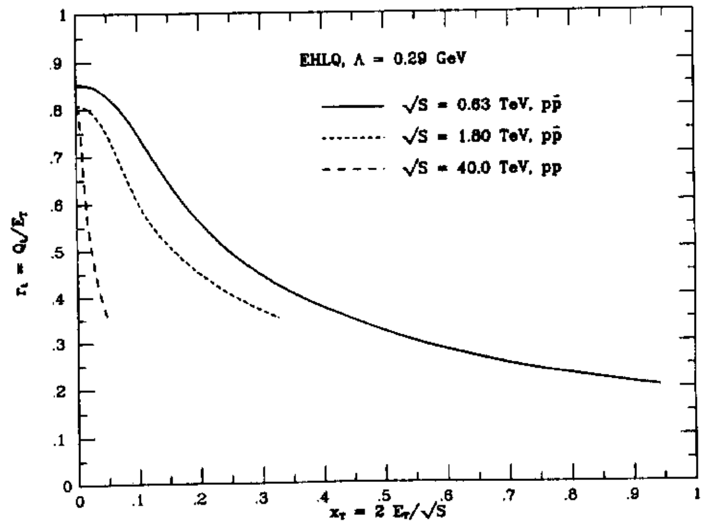

Fig. 5

Results in Figs. 2-4, replotted as $r_t = \langle Q_t \rangle / E_T$ vs. transverse energy fraction x_T, to show violation of the kinematic bound $r_t < 0.71$ at low x_T.

The unphysical results at low x_T imply that at least in this region there must be large higher-order corrections to the predictions we have presented. To compute the order α_S^2 correction to $\langle Q_t \rangle$ one would have to consider not only four-parton final states but also loop corrections to all $2 \to 3$ parton processes, a formidable task. At higher x_T, where the predictions are well within kinematic limits and α_S (E_T) is small, the leading-order results should be more reliable. It would be of interest to compare them there with experiment and with the predictions of Monte Carlo simulations based on leading-pole approximations to QCD.

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